The Scalar-Tensor Inflationary Cosmology

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Abstract

In this paper the scalar-tensor theory of gravity is assumed to describe the evolution of the universe and the gravitational scalar ϕ is ascribed to play the role of inflaton. The theory is characterized by the specified coupling function $\omega(\phi)$ and the cosmological function $\lambda(\phi)$. The function $\lambda(\phi)$ is nearly constant for $0 < \phi < 0.1$ and $\lambda(1) = 0$. The functions $\lambda(\phi)$ and $\omega(\phi)$ provide a double-well potential for the motion of $\phi(t)$. Inflation commences and ends naturally by the dynamics of the scalar field. The energy density of matter increases steadily during inflation. When the constant Γ in the action is determined by the present matter density, the temperature at the end of inflation is of the order of $10^{14} GeV$ in no need of reheating. Furthermore, the gravitational scalar is just the cold dark matter that men seek for.

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I. INTRODUCTION

The idea of inflation [1], as well known, gives not only the solution for the horizon problem and flatness problem that plagued the standard big bang model, but also the explanation of the formation of large-scale structure of the universe from the evolution of the primordial density perturbations. According to current approach, during the evolution of the very early universe, one usually assumes that the energy density happens to be dominated by some vacuum energy and then comoving scales grow exponentially or quasi-exponentially, and the vacuum energy driving inflation is generally assumed to be associated to some scalar field, the inflaton, which is displaced from the minimum of its potential. There results a dilemma. The level of density and temperature fluctuations observed in the present universe, $\delta\rho/\rho \sim 10^{-5}$, require the inflaton potential to be extremely flat, and this is in contrast with the requirement that the coupling of the inflaton field cannot be too small otherwise the reheating process, which converts the vacuum energy into radiation at the end of inflation, takes place too slowly and insufficiently. [2]

The futile effort to identify the inflaton in particle sector and the fact that gravitational force is the only long-range force governing the evolution of the universe lead us to consider the inflation as a pure gravity effect and the gravitational scalar in the scalar-tensor gravity of Bergmann, Nordtvedt and Wagoner [3] as the inflaton field.

In section II, we set down the basic formalism. In the action of the scalar-tensor gravity theory, there are a coupling function $\omega(\phi)$ and a cosmological function $\lambda(\phi)$. The function $\lambda(\phi)$ is nearly constant for $0 < \phi < 0.1$ and $\lambda(1) = 0$. The functions $\lambda(\phi)$ and $\omega(\phi)$ provide a double-well potential for the motion of $\phi(t)$. In the Lagrangian there are three constants ξ, β and Γ in $\omega(\phi)$, $\lambda(\phi)$ and the coupling term respectively. The ξ and β are determined by the large-scale structure data, while Γ is determined by the matter content of the universe, i.e., the temperature of CMB and the density of non-relativistic matter together. We shall have a model with $(\xi, \beta, \Gamma) = (7.5,\ 1.7 \times 10^{-16},\ 0.06362)$. The basic cosmological equations for the Robertson-Walker metric are given in section III.

In section IV we discuss the very early universe. The universe is created from nothing but gravity. After a quantum era, we set the initial condition at the Planck time. The inflation commences naturally because of the pressure p_ϕ of the scalar field ϕ is negative and equals in magnitude with the density ρ_ϕ of ϕ. The gravitational scalar field ϕ is just the inflaton field.

In section V, we discuss the cosmological solutions after inflation. Section VI proposes a realist model. Section VII discusses the primordial nucleosynthesis.

II. THE SCALAR-TENSOR GRAVITY THEORY

We have proposed such a scalar-tensor gravity theory [4]. It is characterized by an action:

$$\mathcal{A} = \int d^4x\sqrt{-g}\,\left[-\phi R - \frac{\omega}{\phi}\phi\cdot\phi - 2\phi\lambda(\phi) - \frac{\Gamma(u\cdot\phi)^2}{1-\phi} + 16\pi L_m\right], \tag{1}$$

in which L_m is the Lagrangian of matter, $\phi\cdot\phi \equiv \phi_{,\sigma}\phi^{,\sigma}$, $u\cdot\phi \equiv u_\mu\phi^{,\mu}$, u_μ is the four-velocity, $\phi^{,\mu} \equiv \partial\phi/\partial x_\mu$, and Γ is a constant of mass dimension 0. The $\Gamma-$term describes the coupling of the field ϕ with matter which is described as an ideal fluid. The coupling function $\omega(\phi)$ and the cosmological function $\lambda(\phi)$ are given as

$$2\omega(\phi) + 3 = \frac{\xi}{1-\phi}\,, \tag{2}$$

$$\lambda(\phi) = 2\xi\beta(1-\phi-\phi\; ln\,\phi), \tag{3}$$

where ξ and β are two dimensionless constants. In a model, we can set $\beta = 1.7\times10^{-16}$ and $\xi = 7.5$ by the data of COBE DMR observations (we shall see in section VI). This value of β is extremely small indeed, yet there is no fine-tuning problem, since it is not a parameter in gauge theory but a number to characterize the cosmological function λ. We note that the function $\lambda(\phi)$ is nearly constant of value $2\xi\beta$ for $\phi < 0.1$ and tend to 0 at $\phi = 1$. That is to say, in the very early universe, there is a very large cosmological constant $\Lambda \sim O(10^{-15})$, but at present day, it will be very small since $\phi \rightarrow 1$. Here we adapt the natural units: $\hbar = c = G = 1$.

Varying action A with respect to $g^{\mu\nu}$, we have field equations

$$R_{\mu\nu} - \frac{1}{2}\,g_{\mu\nu}R = -\frac{8\pi}{\phi}\,T_{\mu\nu} \tag{4}$$

where $T_{\mu\nu}$ is the total stress tensor $T_{\mu\nu} \equiv t_{\mu\nu} + \tau_{\mu\nu}$. The $t_{\mu\nu}$ and $\tau_{\mu\nu}$ are the stress tensors of matter and field ϕ respectively, they read

$$t_{\mu\nu} = p_m g_{\mu\nu} + (\rho_m + p_m)\,u_\mu u_\nu \tag{5}$$

and

$$\tau_{\mu\nu} = \frac{\omega}{8\pi\phi}\left(\phi_{,\mu}\phi_{,\nu} - \frac{g_{\mu\nu}}{2}\phi\cdot\phi\right) + \frac{1}{8\pi}\left(\phi_{;\mu\nu} - g_{\mu\nu}(\Box\phi + \phi\lambda)\right)$$
$$+ \frac{\Gamma}{8\pi(1-\phi)}\left(2u_\mu\phi_\nu u\cdot\phi - \frac{g_{\mu\nu}}{2}(u\cdot\phi)^2\right), \tag{6}$$

where ρ_m and p_m are energy density and pressure of matter respectively. Varying the action A with respect to ϕ, we have field equation of ϕ:

$$(2\omega+3)\Box\phi = 8\pi t^\mu_{\ \mu} + 2\phi^2\lambda' - 2\phi\lambda - \omega'\phi\cdot\phi - \frac{\Gamma\dot{\phi}^2}{(1-\phi)^2} - \frac{2\Gamma\phi\ddot{\phi}}{1-\phi}. \tag{7}$$

III. THE COSMOLOGY: BASIC EQUATIONS

According to the principle of cosmology, $\phi = \phi(t)$. Thus in the Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2sin^2\theta d\phi^2\right), \tag{8}$$

there results the following equations:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{\lambda}{3} + \frac{\omega\dot{\phi}^2}{6\phi^2} - \frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} + \frac{8\pi}{3\phi}\rho_m - \frac{\Gamma}{2\phi}\frac{\dot{\phi}^2}{1-\phi} - \frac{k}{a^2}, \tag{9}$$

$$\frac{\ddot{a}}{a} = \frac{\lambda}{3} - \frac{\omega\dot{\phi}^2}{3\phi^2} - \frac{\dot{a}}{2a}\frac{\dot{\phi}}{\phi} - \frac{\ddot{\phi}}{2\phi} - \frac{8\pi}{3\phi}\rho_m + \frac{\Gamma}{2\phi}\frac{\dot{\phi}^2}{1-\phi}, \tag{10}$$

and

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\dot{\phi}^2}{2(1-\phi)} + \frac{8\pi(3p_m - \rho_m)}{2\omega+3} - \frac{\Gamma}{2\omega+3}\frac{\phi\dot{\phi}^2}{(1-\phi)^2} - \frac{2\Gamma}{2\omega+3}\frac{\phi\ddot{\phi}}{1-\phi}$$
$$= 4\beta\phi(1-\phi^2). \tag{11}$$

The energy density and the pressure of the field ϕ [1] are given by

$$\rho_\phi = \frac{1}{8\pi}\left(\phi\lambda + \frac{\omega\dot{\phi}^2}{2\phi} - 3H\dot{\phi} - \frac{3}{2}\frac{\Gamma\dot{\phi}^2}{1-\phi}\right), \tag{12}$$

[1] With the following expressions of ρ_ϕ and p_ϕ, the two equations (9) and (10) can be expressed as $H^2 = \frac{8\pi}{3\phi}(\rho_m + \rho_\phi) - \frac{k}{a^2}$ and $\frac{\ddot{a}}{a} = -\frac{4\pi}{3\phi}(\rho_m + \rho_\phi + 3p_m + 3p_\phi)$ respectively.

$$p_\phi = \frac{1}{8\pi}\left(-\phi\lambda + \frac{\omega\dot{\phi}^2}{2\phi} + 2H\dot{\phi} + \ddot{\phi} - \frac{1}{2}\frac{\Gamma\dot{\phi}^2}{1-\phi}\right). \tag{13}$$

The energy equation can be found from the identity $(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R)_{;\nu} \equiv 0$, it is

$$\dot{\rho} + 3H(\rho + p) - \frac{\dot{\phi}}{\phi}\rho = 0\,, \qquad (\rho = \rho_m + \rho_\phi, \qquad p = p_m + p_\phi)\,. \tag{14}$$

In turn, the energy equation of matter is given by

$$\dot{\rho}_m + 3H(\rho_m + p_m) = \frac{\Gamma}{4\pi}\left(\frac{\dot{\phi}\ddot{\phi}}{1-\phi} + \frac{1}{2}\frac{\dot{\phi}^3}{(1-\phi)^2} + 3H\frac{\dot{\phi}^2}{1-\phi}\right). \tag{15}$$

The terms in the right-hand side are responsible for the creation of matter by gravitation characterized by the parameter Γ. Heating is mainly in the later period of inflation when $\dot{\phi}$ and $\ddot{\phi}$ get larger. When the ϕ oscillates after inflation, $\dot{\phi}$ and $\ddot{\phi}$ have opposite phases, particle creation is insignificant. The value of Γ is determined by observations of the present universe to be: $\Gamma/\xi = 8.482 \times 10^{-3}$ (see section VI). The temperature at the end of inflation is $\sim 10^{14}GeV$, there is no need of reheating.

From equation (11) we see that the scalar ϕ moves in a double-well potential

$$V(\phi) = \beta(1-\phi^2)^2 \tag{16}$$

which ensures an asymptotic value $\phi_a = 1$ of ϕ. We note that this double-well is provided by the two functions $\lambda(\phi)$ and $\omega(\phi)$ together [2] .

IV. THE VERY EARLY UNIVERSE: INFLATION

A. The creation of the universe

The quantum creation of the universe starts at $t = 0$ when the gravitational scalar $\phi(t) = \phi(0) = 0$. In fact, the probability for creating the universe is given by [5]

$$P \sim e^{-3\phi^2/8V(\phi)}, \tag{17}$$

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[2] It is given by $V'(\phi) = (2\phi^2\lambda' - 2\phi\lambda)/(2\omega + 3)$.

where V is the scalar potential, so, this occurs to the highest local maximum which locates at $\phi = 0$ in the double-well potential. Or, we may argue that [6] the creation of the universe occurs at the zero value of the action (zero action principle), and this is the case $\phi = \partial\phi = 0$ for the action (1).

We are also convinced that the universe is created by gravity from "nothing". Before the creation of universe, there is no any matter but the field ϕ. If the field ϕ takes negative value and is trapped in the left well of the potential, the equation $d^2\phi/d\tau^2 + 4\beta(1-\phi^2) = 0$ has the instanton solution

$$\phi(\tau) = \tanh \sqrt{2\beta}\tau, \tag{18}$$

where τ is the Euclidian time.

At $\phi = \dot{\phi} = 0$, for $k = +1$, the Euclidean equation

$$\left(\frac{da}{d\tau}\right)^2 + \frac{2}{3}\xi\beta a^2 - 1 = 0 \tag{19}$$

has the instanton solution

$$a(\tau) = \frac{1}{\chi} \cos \chi\tau \ , \tag{20}$$

here τ is the Euclidean time. This solution describes the creation of a closed universe from tunneling. At $\tau = 0$, $a = 1/\chi$. By that time, equation (9) has the classical solution

$$a(t) = \frac{1}{\chi} \cosh \chi t \ , \tag{21}$$

where

$$\chi \equiv \sqrt{2\xi\beta/3} \ . \tag{22}$$

B. The initial condition

The universe after its creation experiences a quantum era whence matter is created. As this quantum era is known very little, we shall set the initial conditions at the Planckian epoch t_0 such that classical description is adequate hence forward. At the Planckian epoch, we can adopt the Planck relations: $l_{Pl} = \sqrt{G}$ and $t_{Pl} = \sqrt{G}$, and replace the gravitational constant G by ϕ^{-1}. It is reasonable to let $a_0 \equiv a(t_0) \simeq \chi^{-1}$, then the initial condition follows as

$$t_0 = a_0 \simeq \chi^{-1}, \quad \phi_0 \simeq \chi^2. \tag{23}$$

The value of ρ_{m0} is set to satisfy the causal constraint $a_0 H(t_0) \leq 1$. In general, ρ_{m0} and $\rho_{\phi 0}$ are of the same order. By the way, the exact value of initial ρ_m is of no importance, since it is damped away by inflation.

C. Inflation

In the very early universe, $\phi \sim 0$, we see from eqs. (12,13) that $p_\phi = -\rho_\phi$. At the time t_0, ρ_ϕ and ρ_m are of the same order, there exists $3p + \rho < 0$, therefore inflation commences already. From this time on, ρ_m decreases rapidly by redshift, ρ_ϕ increases and soon dominates, there exists $p = -\rho$, then the universe inflates exponentially with a Hubble parameter:

$$H = \chi_e\ . \tag{24}$$

The constant χ_e is somewhat different from χ given in (18). Since, in the slow-rolling approximation, the term $-H\dot{\phi}/\phi$ in equation (9) is just $-4\beta(1-\phi^2)/3$, so that the $2\xi\beta$ in $\lambda(\phi)$ should reduce to $(2\xi - 4)\beta$, we set it to be $(2\xi - 3)\beta$ to take account of the $\omega\dot{\phi}^2/6\phi^2$ term. Thus we take

$$\chi_e = \sqrt{2\xi_e \beta/3}\ , \qquad \xi_e = \xi - 1.6\ . \tag{25}$$

The solutions of eqs. (9,11) are

$$a(t) = a_0\, e^{\chi_e (t - t_0)}, \tag{26}$$

$$\phi(t) = \phi_0 e^{D(t - t_0)} \tag{27}$$

respectively, where

$$D = (\sqrt{1 + 8/(3\xi_e)} - 1)\sqrt{3\xi_e \beta/2}\ . \tag{28}$$

In the above solutions, we have set $t_0 = 0$. In equation (27), we have deleted the decaying solution. Substituting the solution (27) into equation (15) and integrating, we have

$$\rho_m = \frac{\Gamma r^2 (3H + r)}{4\pi(4H + 2r)} \phi^2(t), \tag{29}$$

we have deleted the decaying solution also. ρ_m increases with ϕ^2 steadily, there is no reheating problem.

When $\phi(t)$ rolls down the potential hill, all of ϕ, $\dot{\phi}$ and $\ddot{\phi}$ increase, the energy density of the field ϕ dominates continuously. When ϕ gets larger than 0.4 or so, $\rho_\phi + p_\phi$ becomes positive and increases as ϕ increases. When ϕ gets to 0.9 or so, it happens $\rho_\phi + 3p_\phi = 0$, it is the same that $\rho + 3p = 0$, then inflation ends naturally. The e-fold number of inflation is thus

$$N = \frac{\chi_e}{D}(ln\,0.9 - ln\,\phi_0). \tag{30}$$

We may compare calculations from the above formulae with computer solutions of the simultaneous equations (9, 11, 15) given in Table 1. Here we set

$$\xi = 7.5\,, \qquad \beta = 1.7 \times 10^{-16}, \qquad \Gamma = 0.06362.$$

The initial conditions according to the above subsection are

$$\phi = 2.55 \times 10^{-15}, \qquad \dot{\phi} = 2.5 \times 10^{-23}, \qquad \rho_m = 10^{-30}, \qquad a = 2 \times 10^7.$$

The calculated Hubble parameter, the ratio $\dot{\phi}/\phi$ and the e-fold number are

$$\chi_e = 2.5859 \times 10^{-8}, \qquad D = 7.9508 \times 10^{-9}, \qquad N = 109.$$

These values calculated from equations (25−30) can be compared with the numerical solutions which are compiled in Table I.

∘ The calculated e-fold number is 109 and that from numerical solutions is 110.

∘ The Hubble parameter H is nearly constant from Hubble time 7 to 90 because of the pressure and the energy density of field ϕ are equal in magnitude and opposite in sign and $\rho_\phi > 10^5 \times \rho_m$.

∘ The value D, i.e., $\dot{\phi}/\phi$, are nearly constant in the period mentioned above, that is to say, ϕ increases exponentially indeed.

∘ The initial ρ_m is damped away by 10 Hubble times. In the period $10 - 90$ Hubble time, the $\rho_{m(cal)}$ is less than that of numerical solutions a little because of we have ignored the second term in the right bracket of equation (15) and $1 - \phi$ is replaced by 1. ρ_m increases with ϕ^2 in this period.

∘ The temperature at the end of inflation is $3.5 \times 10^{13}\, GeV$, this is the highest temperature of the universe. Therefore GUT is never exact and there is no monopole problem at all since monopoles are supposed to be produced when the exact GUT is broken.

Table 1. Numerical solutions of equations (9,11, 15).

t_{Hub}	H	ϕ	$\dot{\phi}/\phi$	ρ_ϕ	ρ_m
1	$1.9429 \cdot 10^{-8}$	$4.9047 \cdot 10^{-15}$	$1.1024 \cdot 10^{-8}$	$3.9893 \cdot 10^{-31}$	$1.7903 \cdot 10^{-32}$
4	$2.4903 \cdot 10^{-8}$	$7.4088 \cdot 10^{-15}$	$8.4789 \cdot 10^{-9}$	$5.8882 \cdot 10^{-31}$	$3.3940 \cdot 10^{-34}$
7	$2.5848 \cdot 10^{-8}$	$3.5084 \cdot 10^{-14}$	$7.9534 \cdot 10^{-9}$	$2.7981 \cdot 10^{-30}$	$6.9693 \cdot 10^{-43}$
10	$2.5848 \cdot 10^{-8}$	$8.8348 \cdot 10^{-14}$	$7.9534 \cdot 10^{-9}$	$7.0461 \cdot 10^{-30}$	$1.7957 \cdot 10^{-45}$
20	$2.5848 \cdot 10^{-8}$	$1.9166 \cdot 10^{-12}$	$7.9534 \cdot 10^{-9}$	$1.5285 \cdot 10^{-28}$	$8.4303 \cdot 10^{-43}$
30	$2.5848 \cdot 10^{-8}$	$4.1549 \cdot 10^{-11}$	$7.9534 \cdot 10^{-9}$	$3.3137 \cdot 10^{-27}$	$3.9622 \cdot 10^{-40}$
40	$2.5848 \cdot 10^{-8}$	$9.0219 \cdot 10^{-10}$	$7.9534 \cdot 10^{-9}$	$7.1953 \cdot 10^{-26}$	$1.8681 \cdot 10^{-37}$
50	$2.5848 \cdot 10^{-8}$	$1.9559 \cdot 10^{-8}$	$7.9534 \cdot 10^{-9}$	$1.5599 \cdot 10^{-24}$	$8.7799 \cdot 10^{-35}$
60	$2.5849 \cdot 10^{-8}$	$4.2402 \cdot 10^{-7}$	$7.9333 \cdot 10^{-9}$	$3.3817 \cdot 10^{-23}$	$4.1265 \cdot 10^{-32}$
70	$2.5850 \cdot 10^{-8}$	$9.207 \cdot 10^{-6}$	$7.9530 \cdot 10^{-9}$	$7.3435 \cdot 10^{-22}$	$1.9453 \cdot 10^{-29}$
80	$2.5872 \cdot 10^{-8}$	$1.9945 \cdot 10^{-4}$	$7.9477 \cdot 10^{-9}$	$1.5935 \cdot 10^{-20}$	$9.1192 \cdot 10^{-27}$
90	$2.6143 \cdot 10^{-8}$	0.00422587	$7.8820 \cdot 10^{-9}$	$3.4475 \cdot 10^{-19}$	$4.0464 \cdot 10^{-24}$
100	$2.7722 \cdot 10^{-8}$	0.0753908	$7.4583 \cdot 10^{-9}$	$6.9145 \cdot 10^{-18}$	$1.2467 \cdot 10^{-21}$
105	$2.7483 \cdot 10^{-8}$	0.76283	$6.9723 \cdot 10^{-9}$	$2.4894 \cdot 10^{-17}$	$1.8678 \cdot 10^{-20}$
110	$1.1340 \cdot 10^{-8}$	0.939783	$2.3104 \cdot 10^{-9}$	$1.4136 \cdot 10^{-17}$	$2.9037 \cdot 10^{-19}$

V. THE UNIVERSE AFTER INFLATION

A. Basic equations after inflation

After inflation, let $\phi = 1 - \sigma^2$. We consider the evolution for large time, $t \gg \beta^{-1}(\sim O(10^{-35})\, sec.)$, $\sigma \ll 1$. Equations (9) and (11) become [3]

$$\begin{aligned} H^2 &= \frac{4}{3}\xi\beta\sigma^2 + \left(\frac{\xi}{3} - 2\Gamma\right)\dot{\sigma}^2 + \frac{8\pi}{3}\rho_m - \frac{1}{a^2} \\ &\approx \frac{4}{3}\xi\beta\sigma^2 + \left(\frac{\xi}{3} - 2\Gamma\right)\dot{\sigma}^2 + \frac{8\pi}{3}\rho_m, \end{aligned} \tag{31}$$

$$\begin{aligned} \left(1 - \frac{2\Gamma}{\xi}(1-\sigma^2)\right)\ddot{\sigma} + 3H\dot{\sigma} + 4\beta\sigma &= 6\beta\sigma^3 - 2\beta\sigma^5 - \frac{4\pi}{3}\sigma(\rho_m - 3p_m) \\ &\approx 0. \end{aligned} \tag{32}$$

The energy density and pressure of ϕ are

$$\rho_\phi = \frac{1}{8\pi}(4\xi\beta\sigma^2 + \xi\dot{\sigma}^2 + 6H\sigma\dot{\sigma} - 6\Gamma\dot{\sigma}^2), \tag{33}$$

$$p_\phi = \frac{1}{8\pi}\left(4\beta(2-\xi)\sigma^2 + (\xi-2)\dot{\sigma}^2 + 2H\sigma\dot{\sigma} - \frac{2\Gamma(\xi-2)}{\xi}\dot{\sigma}^2\right), \tag{34}$$

and the equation (15) becomes

$$\dot{\rho}_m + 3H(\rho_m + p_m) = \frac{\Gamma}{\pi}(\dot{\sigma}\ddot{\sigma} + 3H\dot{\sigma}^2). \tag{35}$$

Since the energy density ρ_ϕ dominates and it is nonrelativistic [4], the universe expands by power law about 2/3. From equation (32), we see that σ oscillates as

$$\sigma(t) = f(t)\cos(\varpi t), \qquad \varpi^2 = 4\beta/(1-2\gamma), \tag{36}$$

where

$$\gamma = \frac{\Gamma}{\xi}\,. \tag{37}$$

By equation (32), $f(t)$ satisfies

$$\dot{f} = -\frac{3H}{2(1-2\gamma)}f\,. \tag{38}$$

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[3] The exact expression is $H^2 = \frac{2}{3}\xi\beta(\sigma^2 - (1-\sigma^2)ln(1-\sigma^2)) + \frac{\xi\dot{\sigma}^2}{3(1-\sigma^2)} - \frac{\sigma^2\dot{\sigma}^2}{1-\sigma^2} + 2\frac{\dot{a}}{a}\frac{\sigma\dot{\sigma}}{1-\sigma^2} + \frac{8\pi}{3}\frac{\rho_m}{1-\sigma^2} - \frac{2\Gamma\dot{\sigma}^2}{1-\sigma^2} - \frac{1}{a^2}$.

[4] When $\phi(t)$ gets near the point 1, the bottom of the potential well, ϕ behaves as particles with effective mass $\sim 2\sqrt{\beta}$ larger than its kinetic energy.

B. The radiation era: $p_m = \frac{1}{3}\rho_m$

By numerical solutions on computer, we see that $f \sim \frac{1}{t}$. Let

$$f(t) = \frac{c}{t}\,, \tag{39}$$

and substituting into (38),we get

$$H = \frac{2(1-2\gamma)}{3t}\,. \tag{40}$$

Then, substituting $f(t)$ and $H(t)$ into equation (35) and integrating, we get solutions:

$$\sigma(t) = \sqrt{\frac{1-8\gamma}{1-4\gamma}}\,\frac{1-2\gamma}{\sqrt{3\xi\beta}}\,\frac{\cos(\varpi t)}{t}\,. \tag{41}$$

The densities are

$$\rho_m = \frac{(1-2\gamma)\gamma}{\pi t^2}\,, \tag{42}$$

and

$$\rho_\phi = \frac{(1-2\gamma)(1-8\gamma)}{6\pi t^2}\,. \tag{43}$$

In the above solutions, the constant c has been determined by equation (31).

C. The dust era: $p_m = 0$

After the time t_{eq}, the time of radiation-dust equality, $p_m = 0$, the numerical solutions show that

$$H = \frac{2}{3t}\,. \tag{44}$$

From (38), we have

$$f(t) \sim \frac{1}{t^{1/(1-2\gamma)}}\,. \tag{45}$$

After integrating (35) and determining the integrating constant, we have solutions:

$$\sigma(t) = \sqrt{\frac{1-8\gamma}{1-4\gamma}}\,\frac{1}{\sqrt{3\xi\beta}}\,\frac{t_{eq}^{2\gamma/(1-2\gamma)}}{t^{1/(1-2\gamma)}}\,\cos(\varpi t)\,; \tag{46}$$

$$\rho_m = \frac{1}{6\pi t^2}\left(1 - \frac{1-8\gamma}{1-2\gamma}\left(\frac{t}{t_{eq}}\right)^{-4\gamma/(1-2\gamma)}\right), \tag{47}$$

and

$$\rho_\phi = \frac{1}{6\pi t^2} \frac{1-8\gamma}{1-2\gamma} \left(\frac{t}{t_{eq}}\right)^{-4\gamma/(1-2\gamma)} . \tag{48}$$

The function ω increases as $\sim t^2$. For the given model, for example, at $t = 1 sec$, $\omega \sim O(10^{73})$. The scalar-tensor gravity agrees with the general relativity exactly.

VI. THE MODEL

Now we determine the three parameters (ξ, β, Γ) in the model from observation data. The parameter Γ can be determined by the values of h, T and Ω_m at present time t_0 as follows.

A. Determining Γ/ξ

We assume $\Omega_0 = 1$ and set $h = 0.6$, then $t_0 = 6.361 \times 10^{60} = 10.87\, Gy$ and $\rho_c = 1.312 \times 10^{-123} = 6.765 \times 10^{-30}\, g/cm^3$. The temperature is determined to be $T_0 = 1.925 \times 10^{-32} = 2.728\, K$ and $\rho_\gamma = 9.039 \times 10^{-128} = 4.662 \times 10^{-34}\, g/cm^3$. The density of matter is assumed to be $\Omega_m = 0.4$ and so $\rho_m = 5.248 \times 10^{-124} = 2.706 \times 10^{-30}\, g/cm^3$. Thus the temperature of dust-radiation equality is calculated to be

$$T_{eq} = 6.651 \times 10^{-29} = 9423\, K. \tag{49}$$

We have the first equation to determine the parameter Γ and the time t_{eq}:

$$\gamma(1-2\gamma) \left(\frac{t_0}{t_{eq}}\right)^2 = 2.747 \times 10^9, \tag{50}$$

where $\gamma \equiv \Gamma/\xi$. On the other hand, relating the expression (47) for $\rho_m(t_0)$ to the value $\rho_m = 1.822 \times 10^{-124}$ gives the second equation:

$$1 - \frac{1-8\gamma}{1-2\gamma} \left(\frac{t_0}{t_{eq}}\right)^{-4\gamma/(1-2\gamma)} = 0.4\, . \tag{51}$$

Solving the above two equations simultaneously gives

$$\begin{aligned} &t_0/t_{eq} = 573900, \qquad t_{eq} = 1.108 \times 10^{55} = 18940\, yr, \\ &\gamma = 0.008482. \end{aligned} \tag{52}$$

It should be noted that this result depends only on the adapted values of h, T_0, Ω_m but not others.

B. The spectra of density perturbations and gravitational waves

Since the energy density of the field ϕ dominates in the evolution of the universe, we consider its adiabatic density perturbations. We perform a conformal transformation:

$$g_{\mu\nu} = A^2(\tilde{\phi})\tilde{g}_{\mu\nu}, \qquad A^2(\tilde{\phi}) = \frac{1}{\phi} \; , \tag{53}$$

and

$$\tilde{\phi} = \sqrt{\frac{\xi}{16\pi}}\, ln\frac{1-\sqrt{1-\phi}}{1+\sqrt{1-\phi}}, \qquad \frac{d\tilde{\phi}}{d\phi} = \sqrt{\frac{\xi}{16\pi}}\frac{1-\phi}{\phi}, \tag{54}$$

then

$$\mathcal{A} = \int d^4x\sqrt{-\tilde{g}}\,[\frac{1}{16\pi}R(\tilde{g}) - \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{\phi}^{,\mu}\tilde{\phi}^{,\nu} - 2\tilde{V}(\tilde{\phi})], \tag{55}$$

where

$$\tilde{V}(\tilde{\phi}) = \frac{\lambda(\phi)}{8\pi\phi} \; . \tag{56}$$

The field $\tilde{\phi}$ is a canonical field, then we can calculate the slow-roll parameters

$$\epsilon \equiv \frac{1}{16\pi}\left(\frac{V'(\tilde{\phi})}{V(\tilde{\phi})}\right)^2 = \frac{1}{\xi} \; , \tag{57}$$

and

$$\eta \equiv \frac{1}{8\pi}\frac{V''(\tilde{\phi})}{V(\tilde{\phi})} = \frac{1}{\xi} \; . \tag{58}$$

The spectrum of adiabatic density perturbation is

$$\delta_H^2(k) = \frac{512\pi}{75}\frac{V^3(\tilde{\phi})}{V'^2(\tilde{\phi})} = 0.03395\,\frac{\beta\xi^2}{\phi}. \tag{59}$$

In these results, we have noticed that $\phi \ll 1$ for interested k in the range $1/h^{-1}Mpc - 1/3000h^{-1}Mpc$.

The spectral index is

$$n_s = 1 + \frac{d\,ln\,\delta_H^2(k)}{d\,ln\,k} = 1 - \frac{D}{H} = 1 - p \; . \tag{60}$$

where $p \equiv D/H = \frac{3}{2}\left[(1+8/3\xi_e)^{1/2} - 1\right]$ which depends on ξ only. We note that $d\,ln\,k \sim Hdt$, since at horizon crossing $k = aH$ and $a \sim e^{Ht}$, H is constant.

Inflation also generates gravitational waves, whose relative contribution to the mean-squared low multipoles of the CMB anisotropy is

$$r_{ts} \equiv \frac{5}{8\pi}\left(\frac{V'(\tilde{\phi})}{V(\tilde{\phi})}\right)^2 = 10\epsilon \ . \tag{61}$$

This more correct factor 10 was given by Polarski and Starobinsky [7]. For our model, $r_{ts} = 1.3$, this rather large value should be tested by future observations.

C. COBE

According to COBE DMR observations of large-angular-scale CMB anisotropy [8], we normalize the density perturbation spectrum using the result [9]:

$$\delta_H(H_0) = 1.91 \times 10^{-5}\,\frac{\exp\left[1.01\,(1-n_s)\right]}{\sqrt{1+0.75 r_{ts}}}. \tag{62}$$

Let $n_s = 0.7$ [5] *i.e.*, $p = 0.3$, then

$$\xi = 7.5 \ , \tag{63}$$

and eqs.(59, 61, 62) determine

$$\beta = 1.7 \times 10^{-16}. \tag{64}$$

In this model, the energy scale of inflation is

$$\tilde{V}^{1/4} = (6.3 \times 10^{16}\, GeV)\,\epsilon^{1/4}. \tag{65}$$

To convince the viability of this model, we calculate σ_8 and the large-scale streaming velocities $\sigma_v(r)$ which are the three dimensional velocity dispersions of galaxies within sphere of radius $rh^{-1}Mpc$ after the data have been smoothed with a Gaussian filter of $12\,h^{-1}Mpc$. The results are given [6] in Table 2.

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[5]Smaller n_s gives smaller (ξ, β) but larger large-scale streaming velocities. For example, $n_s = 0.55$, gives (ξ, β) =(5.5, 1.7×10^{-18}), $\sigma_8 = 0.68$, $\sigma_v(40) = 451\, km/s$, $\sigma_v(60) = 414\, km/s$.

[6]The observational data [10] are $\sigma_v(40) = 388(1 \pm 0.17)\, km/s$ and $\sigma_v(60) = 327(1 \pm 0.25)\, km/s$.

Table 2. Some parameters of the model

n_s	σ_8	$\sigma_v(40)$	$\sigma_v(60)$
0.7	0.69	$357\,km/s$	$309\,km/s$

We note that in calculating the streaming velocities, we have adopted the CDM transition function [11]

$$T(q) = \frac{ln\,(1+2.34q)}{2.34q}\left[1+3.89q+(16.1q)^2+(5.46q)^3+(6.71)^4\right]^{-\frac{1}{4}} \qquad (66)$$

with $q = k/h\Gamma_s\,(Mpc^{-1})$, and $\Gamma_s = \Omega_0 h$, $exp(-\Omega_B - \sqrt{\frac{h}{0.5}}\Omega_B/\Omega_0)$. This manifests itself that the gravitational scalar field ϕ is just the cold dark matter men seek for, of course, there may be other dark matter contained in Ω_m (in this model, $\Omega_m = 0.4$, $\Omega_B h^2 = 0.019$) .

VII. THE PRIMORDIAL NUCLEOSYNTHESIS

Now, we consider the constraint from primordial nucleosynthesis on scalar-tensor theories of gravity. Nucleosynthesis is sensitive to the speed-up factor ξ_n which cannot differ from unity by more than $\sim 15\%$ without over- or under producing ^{4}He [12]. The ξ_n has been derived to be

$$\xi_n = \frac{A(\varphi_n)}{\sqrt{1+\alpha^2(\varphi_0)}} \qquad (67)$$

with φ_n being the value of the scalar field at the time of nucleosynthesis and φ_0 its value today. $A(\varphi) = 1/\sqrt{\phi}$, $\alpha(\varphi)$ is defined by [12]

$$a(\varphi) \equiv ln\,A(\varphi), \qquad (68)$$

$$\alpha(\varphi) \equiv \frac{\partial a(\varphi)}{\partial(\varphi)}. \qquad (69)$$

Here, $\varphi = \sqrt{8\pi}\tilde{\phi}$, since [12]

$$\mathcal{A} = \int d^4x\sqrt{-\tilde{g}}\,\frac{1}{16\pi}\left[R(\tilde{g}) - \frac{1}{2}\tilde{g}_{\mu\nu}\varphi^{,\mu}\varphi^{,\nu} - 2U(\varphi)\right], \qquad U(\varphi) = \frac{\lambda(\phi)}{\phi}. \qquad (70)$$

Therefore

$$\alpha(\varphi) = -\sqrt{\frac{1}{2\xi}}\sqrt{1-\phi}. \qquad (71)$$

At the time of nucleosynthesis, $A(\varphi_n) = 1 + O(10^{-72})$, and at t_0, $\alpha^2(\varphi_0)$ is of order $O(10^{-104})$, then

$$\xi_n = 1 + O(10^{-72}). \tag{72}$$

We see that it makes no significant differences with the standard cosmology.

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